Algebraic Torus Actions: Geometry and Combinatorics
Project Summary

The present project concerns fundamental research in the field of algebraic geometry, a central branch of modern mathematics which has many vital applications in other branches of mathematics, theoretical physics, computational biology, computer science and engineering.

The theory of algebraic varieties with algebraic torus action is a rich research field on the border of algebraic geometry, topology, representation theory and discrete mathematics. The present project aims at extending applicability of methods established in relation to equivariant cohomology, toric geometry and theory of quasi-homogeneous spaces in order to understand the geometry of algebraic varieties with torus action.

The foundations of the topological part of the theory of spaces with algebraic torus actions were laid by Borel and Atiyah, Bott. The Białynicki-Birula decomposition theorem for these spaces put this theory in the context of algebraic geometry. We are primarily interested in the situation when the torus action has a finite number of fixed points and a finite number of one dimensional orbits. This situation has been studied in the context of equivariant cohomology by Goresky, Kottwitz and MacPherson (GKM) and their followers. However, the present topological theory is not satisfactory from the point of view of algebraic geometry. Our aim is to use and extend the present theory of *GKM varieties* in order to obtain answers to problems in algebraic geometry.

As an outcome we expect new results in Minimal Model Program and classification of Fano manifolds. The objects under study will be GIT and Chow quotients, Hilbert schemes and linear systems on varieties with torus action. Moreover, the singularities coming from Fano-Mori contractions, closure of the orbit action strata or degenerations of smooth varieties with torus action will be studied.

Grids, which are refined versions of *GKM graphs*, and *p-divisors*, stemming from toric geometry, will be used to accumulate data about GKM varieties in terms of objects living in the space of characters (or its dual) of the acting torus. Convex and tropical geometry as well as combinatorial methods are expected to provide tools to process and analyze this data. In addition, standard algebro-geometric methods, like those used in the Minimal Model Program, will be applied.